MATHEMATICS TEACHING

THE BULLETIN OF THE
ASSOCIATION for TEACHING AIDS in MATHEMATICS

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MATHEMATICS TEACHING

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No. 8 NOVEMBER, 1958 2/6

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EDITORIAL

In the early days of the Association everybody knew everybody else. Now, with a membership of over 1,000 this can no longer be. The increase in size has brought other changes, some of which have crept on us imperceptibly. The routine tasks of keeping the Association going now take considerable time. Many of our original members in the course of a year travel to meetings in all parts of the country and give talks and demonstrations, and have correspondingly less time for fresh thinking. Now that our organization is becoming settled there is the danger that we are not making sufficient demands on our new members or using their talents sufficiently.

At one time if you joined the Association you were immediately given a job. Now many members join rather hoping that they will be given results and established methods to use. The aim of the Association six years ago was to conduct continuous research aimed at finding what the best methods were, and the attack on the tasks which the Association set itself has still hardly begun. There is urgent need for basic research, for the study of teaching methods and for the development of teaching material. Unless this research is pressed energetically by larger teams of workers there is the danger that our demonstrations will continue in a set pattern, being given by people who are already busy and who have ever-diminishing time in which to carry out fresh research themselves.

Our aim has always been that teachers should find out methods and design their own aids, and pass their experience on to others.

New members are understandably reticent to offer their services, but unless volunteers come forward and take creative tasks upon themselves the Association's advance will slow up because of its ever-increasing inertia. It is now no longer possible for the Committee to know personally each new member, and it is no longer as easy as it was to emphasise the pressing need for people who are willing to take a creative part in our activities.

The rise of local groups is a most encouraging development; the West Riding branch have commenced work on projects concerned with Grammar Schools and Modern Schools, and the programme of the North-Western Group can be seen later in this bulletin. But these activities do not reach every member by any means. Individuals can help in many ways. They can take the initiative in forming groups in their own localities if they are needed, they can write articles for this bulletin, or they can participate in the programmes of work on model and film making which are already under way. There is, for example, great need for a programme of study of the use of film in teaching mathematics at elementary levels—including the production of our own films for the purpose. And if anyone needs further ideas for a programme of work within the framework of the Association then please write to the Editor or the Secretary.

DEMONSTRATION LESSONS

During the last twelve months we have been responsible for Day Conferences at Leicester, Doncaster, Worcester, London, Manchester, Greenford, Saffron Walden, Chichester, Potters Bar and Oxford, each of which has included demonstration lessons followed by discussion. Members would no doubt like to know the reasons for this deliberate policy of the Committee.

To ask what is the purpose of giving a demonstration lesson is at the same time to ask what genuine meeting is possible between man and man. If it is I who give the lesson, can I receive not only the response of my class, but also the reaction of the onlooker? Can the onlookers, individually, experience the unity of what is taking place, entering into it, and then communicating what they met?

Our experience, from giving such lessons at Day Conferences, and otherwise, confirms that these mutual experiences can happen: but it is also clear that a feeling of tension, very uncomfortable, is sometimes a manifest result for some members of the audience. Is it this tension which inhibits discussion and makes for criticism which is obviously emotionally charged? Can those who have experienced in this way tell us whether, eventually, some useful purpose was served in their lives from the fact that they shared this situation of the demonstration lesson?

We are continually asked to declare the *aims* of our lessons. What can we say? "Aim" is a word to be interpreted at different levels. What is difficult to communicate requires that we should discuss first: "what is a lesson?" I find myself unable to answer those who demand that I give a subject title, e.g. "Pythagoras" or "division of fractions": if that was what the children learnt from the lesson, why do members of the audience want it stated? Does a teacher necessarily proclaim a title to the children *before* (or even after) a lesson has taken place; does the giving of a title add anything to what has been learnt?

Can a lesson be defined by what is learnt during its duration? Is a *demonstration* lesson different from a "closed" class lesson only in that there is an added opportunity, namely, for members of the audience to learn, as well as the pupils, but, naturally, to learn something different?

And we, who declare ourselves to be the "ordinary teachers", is it presumption and conceit that we put ourselves on show? Or are we attempting only to "sell" the *aids* which are part of the title of our Association?

What we attempt to show, in the concrete situation of meeting a class of children whom we do not know, is a reversal of the traditional conception of a teacher. The light that beckons us is that *to teach* is an illusion masking the reality, which is that the pupil has the potentiality *to learn*. We ask ourselves, in humility, how does learning take place? How does a child make his own mathematical experience? We think that we find the beginnings of an answer by providing a situation which is rich in mathematics, and by asking questions which direct attention to the potentialities of the situation. We call this the *"Learner-centred Approach."* But for any one lesson this is only a beginning. If by it the pupils engage themselves, in perception and action, wholeheartedly, they seek

and find relationships in freedom of mental work on the situation. In the dialogue with themselves and each other, the direction in which progress is made is decided not by the teacher but by the class.

Naturally we take as much responsibility for our work as do all teachers, and to fulfil it have, beforehand, attempted to master the mathematics we can see in our situation. But we are adults, conditioned and limited by past experience: the child has the innocent mind and sees differently—looking with purity at what for us cannot fail to be structured. It is the structuring which is the experience the child must make for himself. If we are to help him, we can at most be one of his learning aids.

So, my refusal to give a subject title to "explain" my lesson is seen now in a clearer perspective. My *aim* is that the pupils shall have the opportunity to gain mental power: to move forward from where they are by entering in to the situation with which I present them. It is impossible that I should know beforehand what will happen. My intention is to free myself entirely from any such pre-conception, so that my energy will be available to meet what I find before me and to play an active part; looking and listening.

And what, pray, in a given demonstration lesson, did happen? Is it not right that the demonstrator should ask this question of you, the member of the audience? If you took part, you will know. R. M. F.

CHAOS IN ST. LOUIS

The class loves to catch the master out, and the staff in their turn delight when the Head is caught bending. It is a long established tradition that the school timetable is the responsibility of a mathematician; but nowadays the skill of mere human mathematicians is at a discount and everybody knows that machines are so much better. Beaumont High School, St. Louis, U.S.A., have taken the obvious course and used an electronic computer to hand our programme cards and route the students to their correct lessons on the first day of term.

But alas! some boys turned up at girls' gym classes, and we read that even a week later some of the 2,000 students were still wandering around corridors trying to find out where they ought to be. The principal declared that it was the worst muddle that he had seen for 34 years.

The mathematicians must be laughing at the fallibility of machines and Mr. Chips (Senior Maths) can point out that responsibility for the timetable properly belongs to him. After all Mr. Chips knows that no machine is going to put his free periods just where he wants to have them.

The Editor will be pleased to hear from anyone in the London area, or elsewhere, who would like to join a "Working Party" engaged on producing material for this bulletin, or any other form of teaching aid.

A SYSTEM OF NUMERICAL REPRESENTATION BY MEANS OF COLOUR

I. MACFARLANE SMITH

The idea of using coloured materials in the teaching of arithmetic and mathematics is not a new one. For many years, teachers of junior classes have used coloured materials such as beads, blocks and rods, and it is common practice among teachers of mathematics to use coloured chalk for drawing diagrams. The discovery of wide individual differences in imagery has drawn attention to the need for making use of concrete materials as a teaching aid wherever possible, and it is becoming increasingly evident that some pupils not only retain best what they can see and visualize, but in fact do much of their thinking in visual terms. To cater for such pupils, there is an obvious need for devices which make their appeal through the eye rather than through the ear.

Both Seguin and Montessori made use of a series of brightly painted, wooden rods, varying in length from one decimetre to one metre.[1] Dr. Catherine Stern[2] based her system of Structural Arithmetic on the principle of representing the numbers by coloured blocks measuring 1 unit, 2 units and so on. Thus, Cuisenaire[3] is not the first to exploit the attractiveness of coloured materials in his devices for use in the teaching of arithmetic. In addition to harnessing the powerful psychological attraction of bright colours, however, he has also made colour relations an integral part of the learning process in arithmetic.

Cuisenaire believes that teachers present children with abstract words and concepts at too early an age, and he has designed his material to bridge the gap between concrete experience and the abstractions of mathematical thinking. Gattegno regards Cuisenaire's contribution as of outstanding importance. He writes, "I have no hesitation in saying that Cuisenaire's material has solved the problem of teaching arithmetic and should be made widely known, so that there may be an end to the painful struggles of the children who learn and to the frustration of those who teach."

The writer has experimented in teaching elementary arithmetic to young children by means of Cuisenaire's briquettes.[4] On the strength of this experience he is able to endorse the statement by Gattegno "that children who use them thoroughly enjoy their work, and achieve a speed and accuracy surpassing all expectations." He has no doubt that the material greatly adds interest and efficiency to the arithmetic lessons and makes it possible to lay the foundations of mathematical thinking at the elementary stage.

The material consists mainly of a set of coloured rods or briquettes, ranging in length from 1 to 10 centimetres. They have a cross-section of one square centimetre and are coloured according to certain family relationships. The "red" family is composed of rods whose lengths are 2, 4 and 8 cms. (vermilion, crimson and brown respectively); the "blue" family of those whose lengths are 3, 6 and 9 cms. (light green, dark green and blue); the "yellow" family of those whose lengths are 5 and 10 cms. (yellow and orange); the "white" family of the one measuring 1 cm. and the "black" family of the one measuring 7 cms. Thus, the lengths and colours of the rods are as follows:

1 cm	White	6 cms	Dark Green
2 cms	Vermilion	7 cms	Black
3 cms	Light Green	8 cms	Brown
4 cms	Crimson	9 cms	Blue
5 cms	Yellow	10 cms	Orange.

As Gattegno points out, the connections are quite arbitrary so far as the choice of colour for each "family" is concerned; but within each "family" the underlying principle is that where a number is twice another number, its colour will contain the same pigment; vermilion and crimson, crimson and brown, light green and dark green, yellow and orange. Thus, Cuisenaire seeks to exploit two extremely simple processes which are readily understood by quite young children, namely duplication and halving.

It is not the writer's intention to expound the Cuisenaire system further. For a full account of the method, the reader is referred to the booklet by Cuisenaire and Gattegno entitled *Numbers in Colour*[3] as well as to reviews and articles which have appeared from time to time. The remainder of the present article is not mainly concerned with the Gattegno-Cuisenaire method of teaching number. The aim is rather to describe a system[5] by which numbers and colours may be related mathematically and to show how the system may be applied in the teaching of mathematics and science.

If the Cuisenaire material were to become widely used, children so taught would tend to form permanent associations between numbers and colours e.g. 7 would suggest black and 9 would suggest blue. (Incidentally it seems a pity that the poet's number 7 should be the "black sheep".) If the material invented and patented in the United States by Dr. Catherine Stern were to become generally adopted, then children would tend to form the reverse associations, for 7 would suggest blue and 9 would suggest black! In her book *Children Discover Arithmetic* Dr. Stern provides coloured diagrams and photographs of her material. The lengths and colours of the rods of her Unit Box are as follows:

1 unit,	Green	6 units,	Dark Red
2 units,	Violet	7 units,	Light Blue
3 units,	White	8 units,	Pink
4 units,	Brown	9 units,	Black
5 units,	Yellow	10 units,	Dark Blue.

Dr. Stern's choice of colours for the rods of differing length also seems to be quite arbitrary.

In the system described below, there is little that is arbitrary about the choice of colours and they are related in a manner which is self-consistent and systematic. (In passing, it might be pointed out that Cuisenaire's choice of colours is not entirely consistent with his own declared principle e.g. 9 is grouped with the "green-blue" family, yet it is not a double of 3 or 6.)

The objection might be raised that since there are already several systems of number-colour correspondences for teaching number, it may only produce confusion

to put forward another one. The writer, however, is of the opinion that, sooner or later in the interests of uniformity, it will be necessary for an authoritative body to give its blessing to one such system. If the principle of teaching number by means of colour is accepted, then it would seem desirable that all children be taught the *same* associations, so that a particular colour would always suggest the same number, and children will not be confused when they move from one school to another. Now any system of number-colour correspondences would have more chance of being generally accepted, if it satisfied the following two criteria:

1. It should be based on a simple principle, preferably a mathematical one.
2. It should be in harmony with traditional associations, because there seems to be a deep-rooted tendency to attach symbolic or mystical significance to colours and also, though to a less extent, to numbers.

PROPOSED SYSTEM

The system here proposed is based on the analogy between the colours of the spectrum and the notes of the musical scale, an analogy first suggested by Newton in his *Opticks* published in 1704. It makes use of the fact that both colours and notes can be represented by ratios of some agreed standard frequency. We can link the series of natural numbers to the notes of the piano key-board. The higher octaves of any note are produced by vibrations with frequencies which are twice, four times, eight times, etc. the frequency of the original note. We decide that Middle C with its frequency of 256 (i.e. 2^8 vibrations per second) will be associated with the colour red. Then higher octaves C^{I}, C^{II}, C^{III}, etc. which have vibration rates twice, four times, eight times, etc. (i.e. 2^9, 2^{10}, 2^{11}, etc. vibrations per second) will also be associated with the colour red, since they produce perfect harmonies when sounded with the original note.

A note with a vibration rate three times that of Middle C cannot be expressed as one or more octaves above C. It is in fact one octave above G. Hence, to find a colour corresponding to G and the higher octaves of G, we must multiply the frequency of red light by 3/2, and we find that the new frequency is that of green light. Thus G, G^{I}, G^{II}, etc. will be associated with the colour green.

256	384	512	768	1024	1280	1536	2048
C	G	C^{I}	G^{I}	C^{II}	D^{II}	G^{II}	C^{III}
1	3/2	2	3	4	5	6	8
Red	Green	Red	Green	Red		Green	Red

It is convenient for our purpose that the whole of the visible spectrum is contained within the range of frequencies from that of red to that of twice the frequency of red. Or, putting it another way, we can say that the visible spectrum is comprised within one octave between the wave-lengths 7600×10^{-8}cms. (extreme red) and 3800×10^{-8} cms. (extreme violet), the latter wave-length being half the wave-length of extreme red. Thus, we can find a colour of the spectrum corresponding to any ratio between 1 and 2, by dividing the wave-length of red (7600 Ångström units) by the given ratio. Also, since we can convert any positive number into a number lying between 1 and 2 by the simple processes of successive duplicating

or halving, we can find by this means a colour corresponding to any number whatsoever.

As an illustration, we might calculate the wave-lengths of the spectral colours corresponding to the notes of the diatonic scale. We divide 7600 by the successive ratios corresponding to the frequencies of these notes. (Table I)

TABLE I

Notes of Diatonic Scale ..	C	D	E	F	G	A	B	C^1
Ratio of Frequency to that of C	1	9/8	5/4	4/3	3/2	5/3	15/8	2/1
Corresponding Wavelength of Light (in Ångström Units)	7600	6756	6080	5700	5067	4560	4053	3800 or 7600
Corresponding Colours of Spectrum	Red	Ver-milion	Orange	Yellow	Green	Blue	Violet	Red

We could construct a coloured key-board for the piano by colouring the white keys with these seven colours to form a succession of "spectra", each beginning with C or one of its octaves. Different octaves could be indicated by superimposing on the spectral colour cycles a regular gradation from dark to light corresponding to the change in pitch from low notes to high notes. High pitched notes would then correspond to tints such as pinks, pale-yellows, creams, etc. and low pitched notes to shades such as maroons, dark greens, etc. We might go further and colour the lines on the music scores to correspond to the notes, a device which might prove helpful in introducing young children to the early lessons on the piano.

In a similar way, we can find a series of colours to correspond to the series of natural numbers, the colours being arranged in cycles so that halves, doubles, quadruples, etc. of any number will have the same "family" colour as that number. Such a system is shown in Table II.

TABLE II

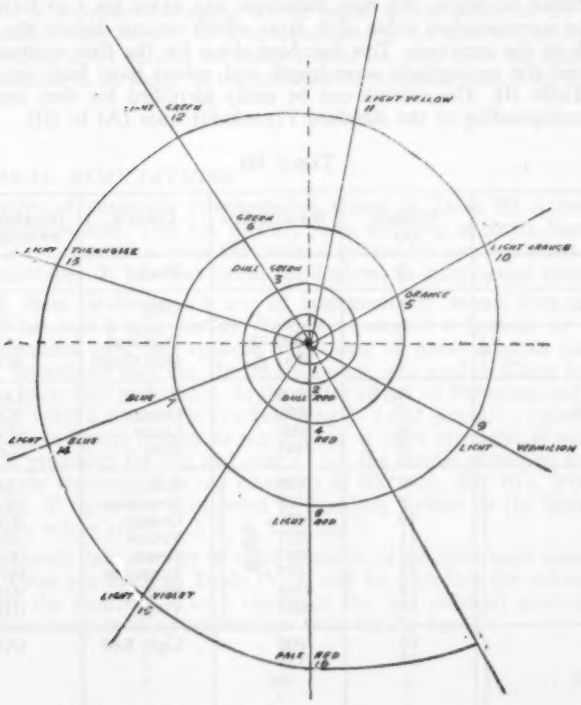
	Red		Orange		Green		Blue	
Fundamental ..	1							
First Cycle	2				3			
Second Cycle ..	4		5		6		7	
Third Cycle ..	8	9	10	11	12	13	14	15
Fourth Cycle ..	16 17	18 19	20 21	22 23	24 25	26 27	28 29	30 31
Etc.	32	36	40	etc.	etc.			

We can calculate appropriate wave-lengths for orange, green and blue by dividing 7600 by 5/4, 6/4 and 7/4 and so on. For violet, we should divide 7600 by 15/8. It is possible, however, to derive a formula giving similar, results and from which we can deduce a wave-length corresponding to any number whatsoever.

THE SPECTRAL MODULUS

The formula $r = 2^{\frac{0}{2\pi}}$ satisfies the conditions. It may be rewritten $0 = 2\pi \log_2 r = 2\pi \log_{10} r / \log_{10} 2$ ..... A.

The curve of this equation is a particular case of the Equiangular or Logarithmic Spiral, a curve with such remarkable properties that James Bernoulli requested



that it be engraved on his tomb with the phrase *Eadem mutata resurgo* ("I shall arise the same, though changed"). One of the properties of the curve is that the radius increases in geometric progression as the angle increases in arithmetic progression. We can arrange that when r takes values 2^1, 2^2, 2^3 or any power of 2 the angle θ corresponds to the wave-length of red light (7600 Ångström units), and for values of r between successive powers of 2, the angle θ corresponds to the wave-lengths of successive colours in the spectrum. We let the initial position of the radius ($\theta = o$) correspond to 7600 Ångström units and the final position ($\theta = 2\pi$) correspond to 3800 Ångström units. As the radius sweeps through 360°, the colour changes from red to violet, repeating the changes in each revolution.

We may think of the equiangular spiral as drawn on a colour circle, the origin being at the centre in a grey region and the radius passing through the colours of the spectrum in each revolution. We may also think of the colours as becoming increasingly bright (i.e. more tinted with white) the further the point travels away from the centre or origin. We may substitute any value for r in formula A and calculate the corresponding value of θ, from which we can deduce the appropriate wave-length in the spectrum. This has been done for the first seventeen cardinal numbers, and the appropriate wave-length and colour have been worked out as shown in Table III. The colours can be easily identified for they are very close to those corresponding to the standard Fraunhofer lines (A) to (H).

TABLE III

	Number (r)	Wavelength	Colour	Nearest Standard Fraunhofer Line
	0		Black	
Fundamental	1		Grey	
First Cycle ..	2	7600 (Dull)	Dull Red	(A) 7594
	3	5067 (Dull)	Dull Green	(E) 5270
Second Cycle ..	4	7600	Red	(A) 7594
	5	6080	Orange	(C) 6563
	6	5067	Green	(E) 5270
	7	4343	Blue	(G) 4308
Third Cycle ..	8	7600 (Bright)	Red	(A) 7594
	9	6756	Vermilion	(B) 6870
	10	6080	Orange	(C) 6563
	11	5527	Yellow	(D) 5893
	12	5067	Green	(E) 5270
	13	4677	Turquoise	(F) 4861
	14	4343	Blue	(G) 4308
	15	4053	Violet	(H) 3969
Fourth Cycle ..	16	7600	Light Red	(A) 7594
	17	7153		
	⋮	etc.	⋮	⋮
	∞	⋮	White	⋮

We could, of course, find wave-lengths corresponding to fractional values of r such as 3/2, or 1/4. 3/2 would correspond to a dark green being half of 3. 1/2, 1/4, 1/8, 1/16, etc. would be represented by a series of increasingly dark reds, maroons, etc. $\sqrt{2}$ would correspond to a dark green and $\sqrt{3}$ to a dark blue. Thus, the scheme makes it possible to construct a wide range of charts (e.g. multiplication tables), in which each number is represented by its colour.

The case for the system rests on its suitability as a device for assisting in the teaching of number relations by means of coloured rods or diagrams. The underlying principle is that families of numbers related by the factor 2 (or its powers) are associated with colours of the same hue, but differing in brightness. This easily-understood principle makes the system helpful for teaching factors, prime factors, aliquot parts, ratio and proportion. It is especially helpful in the early treatment of logarithms, which are now usually introduced by means of a study of the graph of the function $y=2^x$. There is a very simple relationship between logarithms to the base two of numbers belonging to the same family (such as the orange or "golden" family 5, 10, 20, 40 ....). The logarithms to the base two of the members of any such series differ only in the characteristic i.e. they have the same mantissa. Thus, the mantissa is an index of the hue and the characteristic is an index of the brightness.

TRADITIONAL ASSOCIATIONS

The system of numerical representation shown in Table III is based on the binary or dyadic system. This has recently been receiving attention because it has been found more convenient than the decimal system for use in modern electronic computing machines. It has also found applications in information theory.

Though these developments are of comparatively recent date, the binary system itself has had a long history. Leibniz advocated it because he saw in the fact that it requires only two symbols a property of great religious and mystical significance. Something very like the binary system was used in China for purposes of divination from very early times. In the *I-king* (Book of Permutations) which was written before 1,000 B.C., the two symbols yang (–) and ying (– –) were combined to represent eight figures known as the *Pa-kua* or eight trigrams. If we take yang (–, the male principle) for one and ying (– –, the female principle) for zero, the trigrams may be represented in our numerals as 000, 001, 010, 011, 100, 101, 110, and 111. And, if these are considered as numbers written in the binary system, their respective values are 0, 1, 2, 3, 4, 5, 6, and 7.

From an early date a series of eight elements or qualities were attached to the trigrams.[6] These are listed in Table IV. It will be seen that the colours we have proposed for the numbers 0 to 8 (listed in the last column) correspond fairly closely to these long-standing associations with the *Pa-kua*.

TABLE IV

PA-KUA	NUMBER		ELEMENT	PROPOSED COLOUR
	BINARY	DECIMAL		
☷	000	0	Ying, Earth, Darkness	Black
☶	001	1	Yang, Mountain, Meditation ..	Grey
☵	010	2	Water, The Abysmal	Dark Red
☴	011	3	Wood and Wind, Gentleness ..	Dark Green
☳	100	4	Thunder, The Arouser	Red
☲	101	5	Light, Fire, Brightness	Orange
☱	110	6	Low (Marshy ?) Ground, Serenity	Green
☰	111	7	Heaven, Sky, The Creative ..	Blue

It is surely appropriate that zero should be represented by black, since black has natural associations with nothingness and empty space. White has symbolised light and purity and is an appropriate colour for infinity, which might be regarded as a symbol of the Deity. Grey, between white and black, is suitable for unity which is intermediate between infinity and zero. Orange or "gold" is appropriate for the number five which has close associations with the "golden" section. (Five has long been regarded as an important number in life and art. It is the number of sides of the regular pentagon and of the star-shaped pentagram, which has been a symbol of the occult since time immemorial. Each side of the pentagram is cut by the other sides in the divine proportion—the "golden" section.)

The number seven has both sacred and poetical associations. Blue would seem to be an appropriate colour for seven, for in ecclesiastical painting, blue has been used to symbolize piety and divine contemplation, probably because it is the colour of the sky and thus has become symbolic of Heaven.

Green has also been associated with the sacred and holy. Being the colour

of woods and fields it suggests the peace and harmony of nature. Green is perhaps a suitable colour for the number three which has long occupied a special position of veneration and is represented by symbols such as the fleur-de-lis. Green is also perhaps appropriate for the perfect number—six.

APPLICATIONS OF THE PROPOSED SYSTEM

The colour symbolism we have suggested may be used when constructing charts, diagrams or models for teaching purposes. We may indicate simple numerical relationships by means of a suitable choice of colours. Just as we can start with any note on the piano key-board and pick out the diatonic scale in the appropriate key, so we can start with any integer and obtain a complete cycle of spectral colours ending with the integer preceding the double of the first integer. We might think of this on the analogy of changing the key. From our number-colour system, we can obtain a series of colours of any required number and yet forming a complete cycle. Thus, the simplest cycle is 2, 3 (red, green); the next is 3, 4, 5 (green, red, orange). We could have a cycle of seven colours (7, 8, 9, 10, 11, 12, 13) and so on. In choosing a set of colours for a diagram or illustration, it is usually desirable that these be capable of being arranged in a recognized sequence. Such a sequence is provided by the colours of the spectrum. It is also important that the colours should provide the maximum contrast to make them readily distinguishable. If we require only two colours, we should choose the first cycle viz. 2 and 3 or red and green. Since these colours are complementary, they provide the maximum contrast, and presumably this is one reason for the fact that they have been generally adopted for traffic signals. If we require three colours, we should use the cycle 3, 4, 5 (i.e. green, red and orange) which are the colours used in automatic traffic signals. If four colours are needed, we should use the cycle 4, 5, 6, 7 (i.e. red, orange, green and blue). These four colours would be suitable for colouring a political map. (It is believed that *only* four colours are required to show the boundaries between the different countries.)

As a further illustration, let us suppose that we require a series of seven colours, as might happen if we were making a calendar for children, in which the days of the week are to be distinguished by different colours. We should use the colours corresponding to the cycle 7, 8, 9, 10, 11, 12, 13. Thus the colours would be blue (Sunday), red (Monday), vermilion (Tuesday), orange (Wednesday), yellow (Thursday), green (Friday), and turquoise (Saturday).

Our final illustration is taken from the teaching of chemistry. A rational knowledge of chemistry must be founded on a familiarity with the properties of the elements as arranged in the Periodic Table. There are eight main groups of elements, varying in positive valency from 0 to +7. The properties of the elements in these groups show a steady transition as one passes from the group at one end of the table to that at the other end of the table. Thus, the elements in Group I are strongly electropositive, those in Group VII are strongly electronegative. Charts of the Periodic Table could be coloured to illustrate this change in properties. Since there are eight main groups, we should use the colour cycle beginning with 8 (viz. 8, 9, 10, 11, 12, 13, 14, 15) or the eight colours from red to violet. There is a ninth group of elements which do not fit readily into the Periodic Table and which

are among the transition elements. Perhaps purple would be a suitable colour for this group. Purple is in fact a "transition" colour; it does not appear in the spectrum, but completes the colour circle by filling the gap between red and violet.

The colour-scheme indicated in Table V seems to be reasonably appropriate. Vermilion is a suitable colour for Group I which consists of elements producing strongly positive ions, i.e. they are strongly electropositive. Violet is a suitable colour for Group VII which contains elements which are strongly electronegative. There seems also to be a rough correspondence between the proposed colours and the flame colours produced by some of the elements in each group. There are probably sufficient of these favourable associations (some of which are given in Table V) to form a useful mnemonic in memorising the Periodic Table. Diagrams and models of molecular structure could be coloured in such a way that each atom has the colour allotted to the group to which it belongs. The valency of any atom would then be obtained by subtracting 8 from the number corresponding to this colour. Darker colours could be used for the heavier elements in any group.

TABLE V

No.	Colour	Valency	Group of Elements	Associations
8	Red	0	0 Inert gases.	
9	Vermilion	+1	I Alkali Metals.	Lithium salt gives vermilion flame.
10	Orange	+2	II Alkaline Earths.	Calcium salt gives orange flame.
11	Yellow	+3	III Boron, Aluminium, etc.	Boron salt gives yellowish-green flame.
12	Green	+4 or −4	IV Carbon, Silicon, etc.	
13	Turquoise	+5 or −3	V Nitrogen, Phosphorus, etc.	Many phosphates give blue-green flame.
14	Blue	+6 or −2	VI Oxygen, Sulphur, etc.	Sulphur burns with blue flame.
15	Violet	+7 or −1	VII Halogens.	Fluorite shows violet fluorescence.

REFERENCES

(1) Margaret Drummond: *Learning Arithmetic by the Montessori Method.* Harrap, 1947.

(2) Catherine Stern: *Children Discover Arithmetic.* An Introduction to Structural Arithmetic. Harper, 1949.

(3) G. Cuisenaire and C. Gattegno: *Numbers in Colour.* Heinemann, 1956. 2nd Edition.

(4) G. Cuisenaire: Briquettes are obtainable from the Cuisenaire Co. Ltd., 11 Crown Street, Reading, Berks.

(5) This system was suggested by the writer in an article published in *Education Papers,* Journal of the King's College Education Society, Newcastle upon Tyne, Vol. VIII, 3, 1956.

(6) c.f. E. R. Hughes: *Chinese Philosophy in Classical Times.*

SOME THOUGHTS ON THE TEACHING OF MATHEMATICS

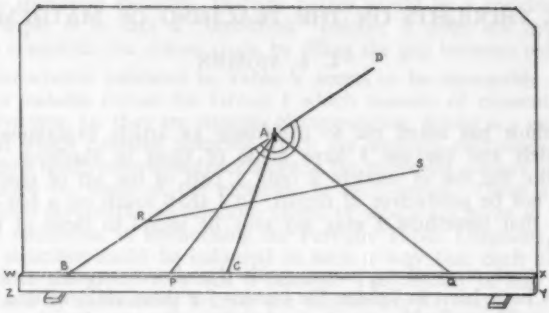
L. A. SWINDEN

Your editor has asked me to contribute an article explaining my aims in making models and the use I have made of them in teaching. This I found difficult to do, the use of models is only a part of the art of teaching, and by itself would not be productive of results. So I shall touch on a few other aspects in the hope that something I may say may be useful to those at the beginning of the road.

I have spent most of my career with the slower streams and my aims in using models have been to lighten the burden for these children and to help them to reach a good creditable standard. Another aim was for the process to be enjoyed both by the children and by me. I have found that the use of models is a tremendous help to the slower child and it does no harm to the quicker ones. And by their use I have often strayed widely outside the examination syllabus and this has acted as a stimulus. The bright colours of many of the models make a strong appeal.

In the teaching of a fresh piece of bookwork in Geometry (for example, the theorem concerning the internal and external bisectors of an angle of a triangle) the work would generally be first taught on the blackboard in the usual way. The diagrams would be large and firmly drawn (some children are many yards away from blackboard) and equal lines would be drawn in the same coloured chalk. (I think colours could well be used more.) Equal angles would be marked a, a_1; b, b_1. This way of indicating equality of angles (numerical suffixes) is a great help in learning, it helps in the solution of riders. It's a great time-saver in writing out and what a help it is in marking! I have not been able to discover who first used three letters for the naming of an angle, but I fancy this was a great disservice to Geometry. In my opinion three letters should only be used when one does not show clearly the angle concerned. After the proof there would be numerical calculations and after mistakes had been discovered the model would be produced and the children would verify the truth of the proposition. This model proved most useful in driving home the case of external division.

The model consists of a piece of plywood supported vertically. In front of this is a length $WXYZ$ of three-eighths wood four inches deep along the top edge of which from B to X is fixed a scale eighty centimetres long graduated in cm. and mm. (part of an old metre rule). A small space is left between this board and the plywood. BAD is a fixed rod with a short break at A to accommodate the movable parts AP, AC, AQ. These are able to turn about A and there is a thumb screw to fix them in any desired positions. AC is the cm. mm. part of a celluloid foot rule. Because of the small space mentioned the position of the side AC of the triangle BAC can be varied. A protractor at A enables the angles BAC, CAD to be bisected by rods AP, AQ and the theorem can be verified in several positions with very fair accuracy. The unwanted parts of AP, AC, AQ are hidden behind $WXYZ$.



A metre rule fixed to ply with sellotape in different positions (at *RS* for example) will provide many more cases. The calculations give practice in handling a slide rule and the right angle *PAQ* links up with the Theorem of Pythagoras, the angle in a semi-circle and the trigonometrical ratios. The more connections between propositions the better grip will the child have. This may be the place to mention that the children have always kept notebooks with skeleton proofs and things such as factor types, steps in solving a quadratic equation by completion of square, and so on. There should be constant revision and testing. This reduces two or three fat text books to a manageable minimum.

Occasionally a model would be used for actual teaching. I have two bits of curtain rail joined at right angles. A protractor can slide on each of these and joining the protractors is a rod whose length between the protractors can be varied. This is used when teaching the trigonometrical ratios. Using this is much quicker than drawing diagrams, the model can be rotated and can be carried to any member of the form needing special attention. I have many times got a thrill when in the course of a first trig. lesson a youngster can tell me the sines of nought degrees and ninety.

I have considerable enjoyment in teaching the idea of a locus. One can sense the increased interest when the form follows the drawing of a cycloid on a miniature blackboard. This has a raised lower portion so that a circular piece of ply with a flange can be rolled along (circumference of locomotive wheel). Then there is a plywood ring fixed on a backboard and circles with or without flange can be rolled on the inside or outside. The straight line resulting from rolling a circle of radius r on the inside of a circle of radius $2r$ always causes amusement. There is a model illustrating a driving wheel, connecting rod and piston, and in the connecting rod two drawing pins so that stages in the transition from circular to linear motion can be seen.

Perhaps it is in dealing with problems in three dimensions that the help given by models is most marked. The model of a tetrahedron (edge fifteen inches) is made of quarter inch dowelling fitting into wooden knobs at joints. The edges are white, two medians of one face (blue and red) support a perpendicular (green) to the opposite vertex. At the foot of the blue median is a median (brown) of

the adjoining face and there is a yellow perpendicular from the vertex opposite to this. In teaching I call the edge two units and I make frequent use of "one, two, root three" and the idea of projection. The description sounds involved, but it is a delight to handle the model and to know that one is "getting over" easily something that is not easy to a child who has difficulty in visualising three dimensions. The skeleton three dimensional models help the children with their drawing of the figures.

Models have been used in work on algebra too. There is a drawing board with graph paper and axes for graphs. A stout wire parabola placed in various positions shows the nature of the roots of a quadratic equation, two real different roots, two real equal roots (never one root), no real roots. Similarly with a cubic equation. There is a rectangular hyperbola and one with asymptotes not at right angles, a circle, an ellipse and a straight line. The latest addition to my collection is a clear perspex cone with some coloured water in it. This illustrates the conic sections in a few seconds. I may mention here that I believe in giving derivations, for example, parallelogram, tangent, secant, parabola, hyperbola, ellipse, and so on. And in teaching stocks the word "par" causes discussion, parabola, paraffin, parhelia.

In the teaching of arithmetic there is not so much scope, I think, at this level; but a collection of valueless share certificates makes the necessary talk in teaching stocks and shares much more real. And I treasure an old £100 bond (apparently valueless but far from valueless from a teaching point of view).

Mensuration is a good topic and a collection of right solids of different shapes is well worth while. All volumes are found by the same method, area of end times length. So areas and pipe line deliveries, railway cuttings and embankments, volumes of tankers, can all be dealt with because they form part of a unified plan. This is good discipline on the importance of care with units. I wonder how many times the models for the volumes of a cone and a pyramid have been used. And I have a cubic decimetre and a litre in a more useful shape (with a handle).

In the preface to *Physical Principles of Mechanics and Acoustics* (a grand book) Prof. Pohl wrote: "Fundamental experiments are brought into prominence throughout, they are intended to make the ideas concerned *as clear as possible to the student.*" (My italics.) Surely in the teaching of mathematics something for the children to see, handle, argue about, would bring benefit and add life to our teaching. The models I use have been on exhibition at many mathematical conferences. Many times the remark has been made, "I wish I had been taught mathematics this way." For examination purposes a good collection of models is a great aid to quick revision. A point too is that for the non-academic child the basic contents of mathematics can be taught and *enjoyed*.

There is a story that Ptolemy once asked Archimedes if there was in geometry any shorter way than that of Euclid's Elements. And the answer was that there is no royal road to geometry. No. But the path can be made smoother and, mixing metaphors, the props can be discarded when they have done their job.

MATHEMATICS IN A MODERN SCHOOL

N. L. KELLETT

Two years ago, when this school was opened, I gave the Head of the Mathematics Department a classroom to develop as a mathematics room. At the same time we decided to try to make the teaching of mathematics in this school as practical as possible. By that I mean not just giving practical examples for children to work, but using pieces of apparatus and models whenever possible to illustrate basic processes and facts. We felt that if we could harness some of the vigour and zest with which pupils approach "practical" subjects, and direct it to other subjects like mathematics, which they did not consider to be practical, then we might get rid of the defeatist attitude which many of them had towards mathematics. In developing this practical aspect, we have gradually built up a stock of models in cardboard, metal and plastic, together with pieces of apparatus, geo-boards, diagrams, etc., many of which are displayed on the walls and shelves of the mathematics room. Pupils can go into this room to examine and use the apparatus and models—and we have found that they are particularly interested in them—so much so that the adjacent room is now being used as a second mathematics room. In fact the two rooms have developed into mathematics "craft" rooms in the same way that we have specialist craft rooms for other subjects. By establishing a Mathematics Department in the building I feel that we have given the subject a permanence and an importance in the eyes of our pupils. Only too often, mathematics teaching is done in any classroom which may be available, and it can very easily come to be regarded as a subject to be pushed into any hole or corner. If subjects like Science, Art and Needlework need special rooms and apparatus, then I think an important subject like mathematics deserves similar treatment. By doing this we have tried to establish in the minds of our pupils that mathematics is a practical subject which they can enjoy and do just like any other practical subject, and that while it is important it is not necessarily difficult. Only by building up such a positive attitude can we destroy the dislike, and in many cases actual fear, of the subject, which so many pupils possess. I feel certain that the majority of our pupils now go to their mathematics lessons with much more confidence in their ability to master the problems they may meet, and with this confidence has come a greater interest in the subject and a liking for it.

As we have four teachers concerned in the teaching of the subject we hold regular meetings to discuss schemes of work and teaching methods. We soon found that confusion was caused for many pupils by different methods of teaching basic arithmetical processes. Not only did we find a variety of methods being taught in different primary schools, but varying methods within the same school. At that time we were receiving pupils from 18 different primary schools, so we invited the staffs of these schools to come and discuss the problem with us. A full and frank discussion took place, as a result of which we reached agreement on uniformity in teaching many of the basic arithmetical processes. While brighter pupils can appreciate that there are different ways of doing some particular process, the less able pupils are often confused by this; particularly when they have mastered the process only after great trouble and effort and I am sure the

uniformity we have now achieved will give further confidence to our pupils. One of the most important aspects of this meeting, however, is the fact that we have been able to meet our primary school colleagues to discuss teaching methods in mathematics. We now have a much better idea of their aims and difficulties and I think they, in turn, appreciate what we are trying to do—and as we are all dealing with the same pupils I am sure the latter are going to benefit from this co-operation.

With the launching of the earth satellites and the resulting pronouncements about the need for more mathematicians and scientists, we felt it opportune to let parents know what we were doing in these two subjects at the school. So we invited them on two evenings, to come to hear about our aims and the methods used in teaching. As far as mathematics is concerned, the Head of the Department talked to parents, who were then able to inspect and use the many aids and pieces of mathematical apparatus which were on view. I think they all realised that not only were we taking the drudgery and monotony out of mathematics, but their children were learning more than they had ever done themselves.

Although we are not doing anything really "new" in our mathematics teaching, I feel that we are achieving something. We have given the subject a high status in the school and we have enlisted the interest and co-operation of the teachers of our future pupils; parents, too, are more conscious of the value of mathematics in the present-day world. In other words, we are building up a tradition of mathematics in the school—that is, a realisation that mathematics is not something difficult and abstract, but a practical down-to-earth subject which most pupils can do successfully. What we want to do is to get our new pupils looking forward to doing mathematics in the same way that they come to the school eager and keen to do Woodwork, Housecraft, Needlework and other craft subjects. If we can successfully induce this attitude of mind into children, then I feel we are on the road to success. Our pupils may not necessarily get higher marks than others in examinations, but I am certain they will get an interest in mathematics and probably a liking for it which will carry them a long way in the future.

MATHEMAGIC

An article by Ann Cutler, under the above title, appeared in last January's *Esquire*, describing the calculation of products by a method in use at the Trachtenberg Mathematical Institute, Zurich. It is claimed that its expert practitioners can beat the calculating machine.

As an example of the method at work, suppose we wish to multiply 7256 by 384. The last digit in the answer is obviously 4. To obtain the last-but-one figure, we mentally add the units of the product 5×4, the tens of the product 6×4 and the units of 6×8. This gives us $0+2+8$ or 10. So 0 is the required digit and 1 is

carried into the next column. Our answer now looks like

$$\overset{1}{0}4.$$

The writer of the article suggests using the middle and forefinger of the left hand, labelled respectively U (for units) and T (for tens), for indicating the relevant figures in the multiplicand and a pencil pointing at the multiplying figure. Some such aid is essential in the early stages of practice to stop the mind wobbling. (Alternatively we can slide a piece of paper with UT written near the edge under the multiplicand.)

Now for the next figure. We will put our UT underneath 25 in the multiplicand and use multiplier 4. Only the units are required from the first product and the tens from the second. Adding these mentally, we move UT one place to the right and multiply the appropriate figures by 8. Lastly (since there are only three figures in the multiplier), we move UT a further place to the right and multiply by 3. Adding in the remainder from the previous operation gives us $8+2+0+4+8+1=23$. That means 3 is our new digit and the 2 is carried.

For the last-but-three figure, we start with UT one place further to the left than at the beginning of the calculation of the previous digit and proceed as before. So there are these three positions to use:

(a) 7 2 5 6 × 3 8 **4**
UT

(b) 7 2 5 6 × 3 **8** 4
UT

(c) 7 2 5 6 × **3** 8 4
UT

Including the carried figure, the total is now $8+0+6+4+5+1+2$, which is 26. Now the answer is

$$\overset{2\ 2\ 1}{6\ 3\ 0\ 4}.$$

The multiplication can be completed by repetition of the procedure, starting with UT one place further left for each subsequent digit. The number of partial products clearly decreases until none of the multipliers has anything left to multiply. At the end, the answer will have been written down as

$$2\ 7\ 8\ \overset{1\ 2\ 2\ 1}{6\ 3\ 0\ 4}.$$

In general, then, put U under the figure in the multiplicand corresponding in position to the required digit of the answer and multiply by the extreme right figure of the multiplier. Shift U one place to the right and simultaneously change to the next figure in the multiplier. Continue until the multiplier is exhausted. With practice it is possible to dispense with the recording of carried figures and with the aid of the letters under the multiplicand.

The advantage of this method over other contractions seems to be that it relieves the mind of the burden of adding numbers greater than ten by considering each product twice.

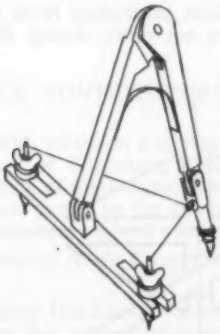
D. H. W.

ON DRAWING AN ELLIPSE

C. BIRTWISTLE

In the number five issue of *Mathematics Teaching* mention was made of compasses for drawing ellipses designed by Senor Fernandez d'Ayamonte and considerable interest has been aroused by this. The principle on which the compass works is not new: a pencil placed in a loop of string which passes around two fixed points (the foci) describes an ellipse. The advantage of the new instrument is that it does this almost automatically and with the ease of an ordinary pair of circle-drawing compasses.

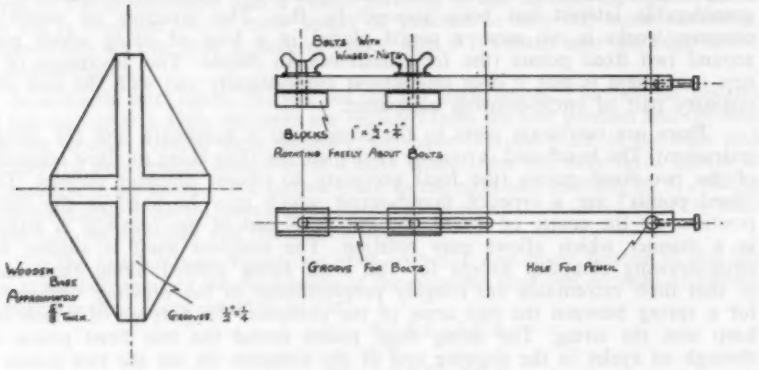
There are two main parts to the compasses: a baseboard and the compass instrument. The baseboard is roughly H-shaped, the slots being to allow adjustment of the two fixed points (the foci) necessary to obtain different ellipses. These "fixed points" are a type of thumb-screw which may be fixed in the desired position. At the centre of the H, the "pointer" end of the compass is attached in a manner which allows easy rotation. The compass itself is similar to a circle-drawing compass, except for the arms being jointed along their length so that their extremities are roughly perpendicular to the drawing surface, and for a spring between the two arms of the compass, the purpose of which is to keep taut the string. The string itself passes round the two fixed points and through an eyelet in the drawing end of the compass. In use the two points are fixed, the string adjusted in length and looped around the fixed points and through the eyelet in the drawing end of the compass, which then draws the ellipse as the compass is rotated.



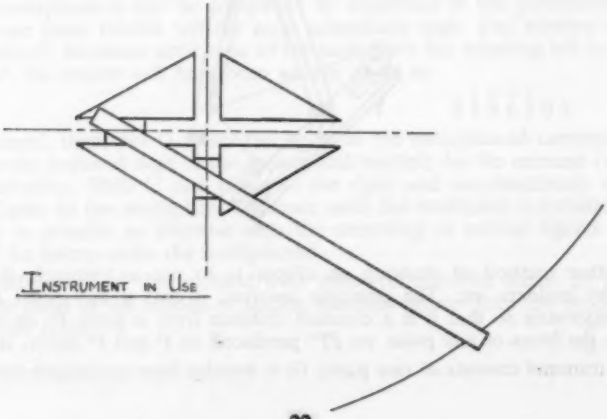
Another method of drawing an ellipse is to use a trammel; this method is used by builders, etc. The principle involved is that if any point P is taken on the major-axis so that it is a constant distance from a point P^1 on the minor-axis, then the locus of any point on PP^1 produced, as P and P^1 move, is an ellipse.

The trammel consists of two parts: (i) a wooden base containing two grooves,

rectangular in section, which are perpendicular to each other and which represent the axes of the ellipse; (ii) a metal bar with a pencil at one end and two adjustable free-moving blocks near the other end. The details of these blocks will be seen from the accompanying diagram, and their purpose is to move freely in the


Bolts With Winged-Nuts
Blocks 1" × ½" × ¾"
Rotating Freely About Bolts
Grooves For Bolts
Hole For Pencil
Wooden Base Approximately ¾" Thick.
Grooves ½" × ¼"

grooves in the wooden base. The adjustment in the slots of the bar allows the two blocks to be moved so that the distance PP^1, which is fixed for each ellipse, may be varied to produce ellipses of different sizes. In actual practice it is advisable to have two short pointed nails protruding from the underside of the wooden block so that the block does not move during the period when the ellipse is being drawn.



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OUR CONTRIBUTORS

Dr. I. Macfarlane Smith, who writes on a system relating numbers and colour, is a lecturer at the Department of Education, King's College, Newcastle-upon-Tyne. He lectures on the teaching of mathematics and physics, but his doctorate is in psychology. He has published papers on the measurement of spatial ability, and constructed selection tests for secondary school courses.

Mr. N. L. Kellett is Headmaster of the County Modern School at Branston in Lincolnshire.

Mr. L. A. Swinden is a retired Headmaster who is well-known in the Association for the prominent part he has played in many exhibitions of models and teaching apparatus.

Mr. C. Birtwistle, of the Secondary Technical School, Nelson, has arranged many of the Association's activities in the North-West, and he took a leading part in founding the local group there.

CONFERENCES

The Autumn season opened on October 4th with the invitation of the Middlesex Branch to all members to visit Mountgrace Comprehensive School, Potters Bar.

On the same date there was a meeting on work in the Primary School, at Preston.

These were followed by a Day Conference on October 11th at Wood Farm County Primary School, Oxford, organised for us by the Institute of Education, and with the title of *Mathematics in the Primary School.*

The meeting of the West Riding Branch was held in Doncaster on October 18th, dealing with work for the third year of the Secondary School.

The North-Western Group held their meeting on October 25th at The School of Education, Manchester University.

On November 8th a Conference was held at Dowsett School, Southend-on-Sea, requested by members in S.E. Essex who want to form a local group.

Forthcoming Meetings

The last meeting of 1958 has the title *Mathematics for Juniors and Seniors,* and the discussions will take place at Ashmead School, Northumberland Avenue, Reading, on Saturday, November 29th.

The Sixth Annual General Meeting of the Association will be held in Birmingham on February 14th, 1959, and will take place during an Open Conference at the Martineau Teachers' Centre, Bristol Road. Members of the Birmingham Branch of the Mathematical Association have joined forces with local members of A.T.A.M. to organise this major event of our year's programme, and the Chief Education Officer has also promised his support. We hope that members who do not live near London will welcome this move to hold the Annual General Meeting in other centres, and that as many as possible will keep the date free, and come to the meeting.

Members will like to know that the Association has been invited to take part in the Educational Exhibition at Olympia, May 25th to June 4th, organised by the National Union of Teachers. It has been agreed that we shall give demonstration lessons.

The Director of Extra-Mural Studies at the University of Liverpool has made a provisional announcement about a Conference which is to be held from April 5th—11th, 1959, at Derby Hall, Liverpool. It will be devoted mainly to lectures and discussions on the uses of mathematics in industry and the possible bearings of these uses on the teaching of mathematics in Grammar School, Technical Schools and Universities. There will be morning sessions in the University and afternoon and evening sessions at Derby Hall. The fee for full membership of the Conference (including residence) will be £15 15s. 0d. but, thanks to the financial assistance promised by a number of industrial concerns, it will be possible for teachers to be admitted at the nominal charge of £3 3s. 0d. Teachers may also enrol as non-resident associate members, eligible to attend the morning sessions at a fee of 10/-.

MATHEMATICS AT LOUGHBOROUGH

The Leicestershire Education Committee ran an ambitious Residential Maths Course for teachers in Secondary Schools from April 13th—19th at Loughborough Training College. The Course consisted of lectures, discussions and practical work. The lectures and discussions were concerned with Maths teaching up to G.C.E. "0" level. The practical work was divided into six groups as follows, each student attending two out of the six:—

Surveying; Practical Approach to Trigonometry; Solid Geometry—the making of models; Navigation, Astronomy and Spherical Geometry; Practical Drawing and Mathematical Applications; Computation and Graphical Work.

Many well-known mathematicians were resident as lecturers or tutors, or both. Exhibitions of various mathematical models were mounted by Mr. L. A. Swinden, Mr. J. W. Peskett and Mr. and Mrs. A. Ivell, and these proved to be a helpful adjunct to a Course that was voted a great success. R. P.

WEST RIDING GROUP

The West Riding Group of A.T.A.M. held a meeting in Don Valley High School, Doncaster, on Saturday, May 17th. In the morning the main topic was a talk by Mr. B. Atkin on models in solid trigonometry. He spoke of the experiments that he had devised which had taken him from the construction of frame models to the making of 3-D film. He projected one of his own film-strips, and using 3-D spectacles members were able to judge for themselves its considerable potentialities. A brisk discussion followed, which ranged from the difficulties of the pupil in three-dimensional visualisation to the problems of stereoscopic photography

After lunch in the school dining hall and a look round the new building the group settled down to discuss the teaching of arithmetic in primary schools. The topic was dealt with in turn by teachers in primary, selective and non-selective secondary schools and gave rise to much discussion.

It was decided during the day to form three sub-groups, one for each type of school, to promote discussion and the preparation of teaching experiments and investigations. Any members in the West Riding who are interested in participating in these activities and who are not already in the group should contact the Secretary, Mr. R. W. Shaw, 23 Pipering Lane, Scawthorpe, Doncaster.

R. W. S.

The proposed course for Primary School teachers organised jointly by the Institute of Education, Sheffield, and the West Riding Branch of the A.T.A.M. has had to be deferred until the Spring Term as it has proved impossible to arrange to book a suitable speaker on one of the important topics in the course. For further details apply to the Secretary, Institute of Education, Sheffield.

NORTH-WESTERN GROUP

Meetings at School of Education of Manchester University, Dover Street, Manchester (opposite main University buildings in Oxford Road).

Saturday, October 25th, 1958. 10.15 a.m. to 4 p.m.

Talk by Mr. R. H. Collins, B.Sc., Chairman of the Association: *The Beginnings of Algebra.*

Discussion on *Difficulties in Secondary Modern School Mathematics,* introduced by a group of speakers.

Saturday, February 7th, 1959. 10.15 a.m. to 4 p.m.

Discussion on *Trigonometry,* opened by Mr. A. D. Walters, M.A., Padgate Training College.

Display of surveying equipment and work done by boys aged 14 to 16 years.

Demonstration and discussion of mathematical film-strips.

May, 1959. Date to be fixed later.

Special meeting on some major aspect of mathematics teaching.

Details available later.

N.B.—All meetings are open to all teachers, whether members of the Association or not; there are no booking arrangements or fees, but a collection is made to defray expenses. Also those intending to be present are asked to notify Mr. Birtwistle within five days of the meeting so that suitable arrangements can be made for accommodation and lunch at the University refectory where required.

All Enquiries, etc., to:—

Mr. C. Birtwistle, 1, Meredith Street, Nelson, Lancs.

PROBLEMS

At St. Andrews from August 5th to 12th, the *International Commission for the Study and Improvement of the Teaching of Mathematics* held its twelfth meeting, attended by 42 people from nine countries.

The spirit prevailing at these conferences is not at all that of the usual course in which some come to lecture and others to listen. Nor is it a question of scoring debating points or of arriving at accurately formulated statements and definitions. If it is possible to state, positively, what this means, the word that comes to mind is *unity*. The least as well as the greatest amongst us know that it is true, when Dr. C. Gattegno formulates it for us "We seek the unknown; this is an initiation into new paths and explorations". What we had become more aware of during the discussions, although it was not specifically mentioned, was of the fragmentation of individual lives and of *human life*. For a little while we experience a unification of our own selves which was unfamiliar to us, and a harmony within the group born of our earnest and felt desire to move forward.

It was clear from the concluding remarks of each participant that almost all had been conscious of a lived experience the depth and results of which they could only guess at. Many expressed their belief that it would take them many

years to know its full significance, and still more that it must and would be put into their lives and could be tested and proved only by its effects on their work and their pupils.

"Problem" had been widened to include lived life. It was seen that a problem, with its attendant anxieties, implied the need to rise to a new level, and that from such transcendence the problem would be found to have disappeared and an unequivocal solution become apparent which could be formulated in techniques. Classifications of types of problems and of what was meant by problem were not difficult to make: at its most general we could say that we were concerned with the part played in mathematics teaching by what was extra-mathematical. It could, of course, include what is called "practical," "everyday" or simply "physics", but of greater significance was the entry of *affectivity* into the teaching situation.

The understanding of this word affectivity, difficult for English ears, brought out another advantage of an international conference: in striving to understand languages other than one's own, enrichment of ideas comes. "*Affectivité, c'est la vie*", we were told, and so we began to see that we had arrived, in simplicity of spirit, at what the psychologists have been trying to make clear for fifty years or more—that it is not the intellect which provides the driving force of our lives, but our feelings. The reason why there is no transference of learning from one subject to another is because the lessons are not of lived experience for the pupils; the whole life is not engaged in the activity, what is gained is factual knowledge which falls short of true understanding.

Those participants who preferred to grasp at concepts more obviously related to the day-to-day work in the schools had their opportunity in group work: beginnings of algebra: "open" problems; continuity in the secondary school course, were some topics dealt with in this way. For the rest of us, the discussion was on "shock affective", and although words fail to say what we achieved, we have certainty that it will prove itself in our lives. Prof. G. Choquet expressed the feeling of all in declaring that he had worked very hard, seeking to master unfamiliar ideas.

It is not an end, but a beginning, and finally we were posed with a special problem to consider: "How does personal time structure itself into public time?"

How many of us will have found the answer before we meet in Lisbon next year to discuss the transition from the Secondary stage to the University?

R. M. F.

Professor W. W. Sawyer, well-known as the author of *Mathematician's Delight* and other books, will be visiting England next summer. He hopes to arrive by June 14th, and offers to do some part-time teaching (unpaid) in a school or other organisation which will offer him some regular cricket in exchange. Anyone interested in this tempting offer should write to him at The Mathematics Department, Wesleyan University, Middletown, Connecticut, U.S.A.

CORRESPONDENCE

Dear Sir,

I should like to express my appreciation of the article "Mathematics with Backward Boys," by S. Hodgkinson, in the July, 1958, issue of "Mathematics Teaching," particularly the measured results of negligible improvement in numerical work when taught carefully to Fourth Form boys (assumed Fourth Year). Many will have arrived at this conclusion, but will not have had the opportunity, etc., to exhibit the facts so clearly.

The only sort of teacher who is able to get really away from arithmetic is either a mathematics specialist or one who has a keen interest in the subject. Even in a district like mine, favoured by applicants for posts, such teachers are just not available, and those who have perforce to take mathematics with the D stream are unable or unwilling to give much thought to graphs or to spatial ideas. A little model making does go on in the earlier years, but even scale drawing is neglected, owing to the disinclination of the teachers concerned. It appears to me that teachers who are both able and willing to teach the D stream really well are much too valuable to be used with those children : the same gifts would bring so much greater return with brighter pupils. Not till there is an ample supply of mathematics teachers can good ones be spared for the D stream.

As far as Mr. Hodgkinson's final dilemma between change of topic and practice required to learn a topic thoroughly, I suggest the solution:—

Teach topic A as far as practicable: move on to topic B but keep A alive by revising with *easy* numbers. When topic C is in hand, revise both A and B with easy numbers, if possible by "mental questions" filled out, where necessary with basic topics. There is an optimum period for the frequency for keeping ideas fresh—say one week or half a week—to be found by experience.

Yours truly,

L. G. E.

There is a fashion now for children's books containing collapsible paper models which open out as the pages are turned. Greetings cards are also constructed on the same principle. These ingenious models depend on a few fundamental linkages and illustrate important ideas in geometry and mechanics.

Could not a book on geometry be made like this? The idea is not new. *The Elements of Geometrie of the most auncient philosopher Euclide of Megara faithfully (now first) translated into the Englishe toung* (Billingsley's translation of 1570) contains nets of pyramids on various bases stuck to the pages which could be folded up to form solid figures; and some later books have no doubt been made in this way, but the method does not seem to be in current use. In any case the interest in the modern examples is in the movements and in the deformations which the figures undergo.

We invite news from your School Maths Club on the theory of greetings cards, or suggestions for a truly mathematical book to be illustrated with this type of model.

BOOK REVIEWS

LE MATERIEL POUR L'ENSEIGNEMENT DES MATHEMATIQUES.—C. Gattegno, W. Servais, E. Castelnuovo, J. L. Nicolet, T. J. Fletcher, L. Motard, L. Campedelli, A. Biguenet, J. W. Peskett and P. Puig Adam. Delachaux & Niestlé, Neuchatel; pp. 212; 37/-.

Mathematics is a deductive logical system based on postulated relations between undefined entities. Because its primitive concepts are undefined, modern mathematics has the great generality that accounts for so much of its usefulness. But the ability to reason accurately about general concepts is now possessed by only a few adults and by even fewer children. Nevertheless, the coming of the age of automation and nuclear energy has made it necessary for large numbers of people to acquire usable skill in mathematics. Some means must therefore be found of enabling people of average capacity to raise their thinking to this new level. It is to such means that this volume, their second, has been devoted by the International Commission for the Study and Improvement of the Teaching of Mathematics.

The book begins with a preface in which the committee introduces the contributors to the reader.

Part I follows. It consists of three chapters dealing with the necessity of providing a wide variety of models from which the pupils can abstract the relations that will later serve them as their mathematical postulates. C. Gattegno gives evidence from the psychology of perception in support of this thesis. W. Servais shows how, by the aid of verbal and other signs, the abstractions of one level may become models for another. E. Castelnuovo emphasizes the importance of providing working models and models usable by the pupils in addition to fixed models that can only be looked at.

One way of introducing models is to show mathematical films. These are still unfamiliar to many teachers, so they have been treated rather fully in the four chapters of Part II. J. L. Nicolet, the Swiss pioneer, explains how films may be used not only to give awareness of the beauty of mathematics and to awaken intuition, thus aiding the making of discoveries, but also to give rigorous proofs. T. J. Fletcher develops this latter idea by pointing out that the changes in the diagram on the screen can correspond to the steps of a reasoned verbal argument. He also gives technical advice that will be most helpful to those thinking of making these films, though, as L. Motard reminds us, such a project is not to be entered on without counting the cost. L. Motard's technical advice supplements that of T. J. Fletcher. C. Gattegno concludes Part II by showing that films may be used in many ways in mathematics teaching, and that they can provide help unobtainable from any other teaching aid.

Nevertheless, mathematical films are expensive, their making is very time-consuming, and adequate facilities for projection are not always available, so teaching models are dealt with comprehensively in Part III, which consists of the last four chapters of the book. M. Campedelli, after discussing the use of models, describes, with several illustrations, those produced by the Institute of Geometry

at Florence. A. Biguenet then gives us a most useful chapter describing the construction of easy to make working models illustrating loci, envelopes, and transformations. J. W. Peskett follows this up by telling us how to handle without mishap all the things required for model making, from balsa wood, plastics and aluminium, to glue and tools. P. Puig Adam, in contrast to this, points out how many of the objects of everyday life can be used not only to suggest mathematical ideas, but also to provide mathematical activities for our pupils. C. Gattegno completes this part by discussing two multivalent teaching aids, the Cuisenaire rods and his geo-boards.

The book ends with an appendix by P. Puig Adam describing a lesson on arithmetical progressions given by means of the Cuisenaire rods, and a list of recommended mathematical films, an index, and a table of contents.

Not only have the authors from seven different European countries written on topics of which they have expert knowledge, but the general outlines of the book were planned in advance. The work has thus achieved a unity that collections often lack. All mathematics teachers who think seriously about their work will be grateful to the authors for having given to the world the benefit of their experience.

C. A. WINYARD.

TIME FOR SUMS.—Elizabeth Taylor. Oliver & Boyd, 1958. Book One, 48pp., 2s. 6d. Book Two, 48pp., 2s. 6d. Teacher's Manual (with answers), 16pp., 1s. 6d. Individual Cards, 35, 2s. 6d. 22 Sum Cards for Book One, 1s. 6d. 28 Sum Cards for Book Two, 1s. 6d.

Time for Sums is another series for the infant school child. There is no reading matter at all. The design of these, the first two books of a series of four, is to exercise the partitions of the numbers 0 to 10. All possible partitions are given with counter patterns, coloured to help the child to structure the way in which the number is broken down. For example, 4 is illustrated on one page as three red circles and one grey one, so that the results of $3+1$, $1+3$, $4-3$, $4-1$, are immediately perceptible. Book One deals with the numbers nought to six and Book Two with seven to ten. It is refreshing to see the addition and subtraction combinations taught together and combined with examples of the type, $3 + ? = 1$.

The individual cards are reproductions of the counter patterns in the books with the number symbols on the back giving all partitions of each number. They may be used as flash cards, game cards or as aids for the sum cards which provide practice in the addition and subtraction combinations.

The cards and the books would provide admirable supplements to the usual kind of experimental work which children undertake and would ensure that the whole range was adequately covered.

The Teacher's Manual is an explanation of the cards and books. Most teachers will not need it. They will use the cards alongside the practical work, which is being done, to help in the development of the concept of number.

Many teachers of the very dull children in the Secondary Modern School may find these cards useful.

C. H.

NUMBER GAMES AND PROJECTS.—T. G. Monks. Oliver & Boyd. 60pp., 4s. 6d.

This is another book of number games and projects. It will be found useful as a compendium of devices for the young student in training. It contains over 50 ideas for games with number, counting devices, drill and practice cards and shops. From this point of view the book fulfils its purpose.

It is a pity that some of the archaic forms still persist. This book still persists in the "Teaching of NUMBER." "NUMB" means "deadened, deprived of sensation, stupefied, motionless!"I often think that the "NUMBER" of these exponents of infant teaching is the comparative form. We must remember that these games are adjuncts to the teaching of mathematics by concentrating on the dynamical quality of mathematical operations.

C. H.

PUZZLE-MATHS.—George Gamow and Marvin Stern. Macmillan and Co. 119pp., 8s. 6d.

You must go out and buy a copy of this for yourself and for the school library. It is a most entertaining book, very easy to read and sufficiently flattering to the reader to include the solutions as conclusions to the puzzles which form the beginning of the stories. Any boy doing O level will find it well within his comprehension and most stimulating to him. Many teachers will find inspiration for the beginning of a lesson.

There are some old favourites here, some new graphical ones and some new and old problems about speed. It would be unfair to the authors to tell you what they are. All the old ones are in a new guise with new twists which make them fresh to the reader.

Well done, Messrs. Macmillan! Let us have more books like this at this price.

C. H.

ARITHMETIC TESTS FOR FOURTH-YEAR JUNIORS.—Kenneth Anderson. Geo. G. Harrap & Co. 111pp., 4s. 6d.

There are many occasions when a test is a good way of finding out what one has not done enough about.

This book is divided into three categories: Mental Arithmetic, Mechanical Arithmetic and Problems. The contents are most comprehensively catalogued and analysed for the benefit of the busy teacher.

When one looks through this book, which I am sure a majority of teachers will find valuable to supplement their present courses, one feels very depressed. Why do we spend so much time on dividing 49 gal. 2 qt. 0 pt. by 9; reducing £12 9s. 5½d. to halfpence? Why must there be so much commercial arithmetic? Think of the child who is confronted with a man delivering 8 mattresses and demanding £23 19s. 4d. each for them! Cannot we find some mathematics for them to do? Does mathematics deserve any better treatment than it gets when this travesty is paraded before the most intelligent of our children?

It is not Mr. Anderson's fault, but his book will perform a valuable service if, in its most comprehensive and admirably constructed tests, tests which fulfil their purpose one might say "almost to perfection," it shows up the futility and the triviality of the mathematical aims of so many of our Primary Schools.

C. H.

THE METHODS OF PURE PROJECTIVE GEOMETRY.—J. Heading. Macmillan. pp. 254 plus ix; 10s.

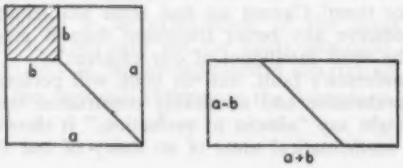
This book is a practical contribution to the teaching of a part of geometry which students find particularly difficult. Its aim is to assist the student to transfer his thoughts from metrical concepts such as length and angle to projective concepts such as cross-ratios and homographies. The method followed is to develop metrical theorems and their projective counterparts side by side, in two distinct strands with alternate paragraphs labelled "M" and "P" which are independent logical entities, but which illuminate one another.

Some previous knowledge of cross-ratios is assumed, but a chapter of revision notes gives a concise summary of what the student is expected to know already. There are already excellent books dealing with co-ordinate projective geometry, and in this book the emphasis is all the while on the understanding and systematic use of pure methods.

Pure projective geometry is notorious for its logical subtlety, and the author quite frankly cuts a Gordian knot by assuming the previous establishment of a projective parametric system on the line. This matter is discussed in detail in an appendix. Also the introduction of complex points into geometry is only mentioned very briefly, and the reader is referred elsewhere for details. This procedure is obviously unsatisfactory from an advanced point of view, but it is almost certainly the wisest course to adopt with pupils who are doing the work for the first time, and who need to acquire some familiarity with the ideas before they can hope to appreciate the finer details of the logical foundations.

The relationship between metrical and projective geometry is well brought out, and within the author's self-imposed limits the treatment is rigorous, detailed, and free from the loose writing about "projecting to infinity" which mars some other books. There is a large collection of problems with hints for solution, and the book should certainly be considered for use by scholarship students and those at the beginning of a University course. Nevertheless, we still await a book covering a wider field that makes the essentials of the subject clear to a beginner. Books on projective geometry seem to have lost sight of its origin in experience. Metrical geometry is the study of things as they are, projective geometry the study of things as they appear. Can no book start there? T. J. F.

MATHS WITHOUT WORDS



ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

CONSTITUTION

1. The name of the Association shall be "THE ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS."

2. The AIMS of the Association shall be to promote the study of the teaching of mathematics, and to improve the aids used therein, through week-end courses, exhibitions, demonstration lessons, and the publication of a bulletin.

3. MEMBERSHIP of the Association is open to all who are interested in these aims.

4. The SUBSCRIPTION shall be five shillings per annum, covering the calendar year, and shall be payable to the Treasurer from the beginning of the year. Subscriptions of new members joining after 1st October shall also cover membership for the following calendar year.

5. The COMMITTEE shall consist of a Chairman, Secretary, Treasurer and nine other members, and shall be responsible for all day to day management of the Association. It shall have power to co-opt additional members, to appoint assistants to Secretary and Treasurer, an Editorial staff for the bulletin, a Librarian, and to delegate responsibility to sub-committees set up to act for particular aspects of the work of the Association.

6. The Committee of the Association shall call an ANNUAL GENERAL MEETING once in the course of each calendar year, and at this Meeting the Committee for the subsequent year shall be elected. The Annual General Meeting shall also have power to fill the office of President for the ensuing year and to elect Vice-Presidents. Such Officers are to be ex-officio members of the Committee. Nominations, duly proposed and seconded and with the consent of the nominee, must reach the Secretary, in writing, at least 28 days before the date of the Annual General Meeting.

7. EXTRAORDINARY GENERAL MEETINGS may be called at any time, either by the Committee, or at the written request to the Secretary of at least twenty members of the Association, with reasonable time allowed for making the necessary arrangements.

8. A QUORUM for a General Meeting shall be 20 members.

9. The CONSTITUTION may be amended by a resolution passed by a simple majority at an Annual General Meeting. Notice of any such resolution must reach the Secretary in writing at least 28 days before the date of the Meeting.

ASSOCIATION FOR TEACHING AIDS IN MATHEMATICS

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The Annual Subscription to the Association is 10s for 1959 and is due on 1st January.; cheques should be made payable to "A.T.A.M", and should be addressed to the Secretary. This journal is supplied free to members.